

RAN-2203080301030002
M.Sc. (Sem. - I) Examination November - 2025
Mathematics (Topology- PGMTH - 103)
Time: 3 Hours]
[Total Marks: 70
સૂચના : / Instructions

- (1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the above details on your answer book
- (2) Answer all questions
- (3) Figures to the right indicate marks
- (4) Follow usual notations and conventions.

Seat No.:

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Student's Signature

- Q. 1 Attempt Any Two: 14**
- a. For a non-empty set X , let x be a point in X and let T_x consist of empty set $\emptyset$ and all those subsets of X which contains x . Show that T_x is a topology on X .
 - b. Let X be a topological space and Y be a subspace of X . Then show that subspace S of Y is closed in Y if and only if there is a closed set F in X such that $S = F \cap Y$.
 - c. Prove that a subspace of a topological space itself a topological space.
- Q. 2 Attempt Any Two: 14**
- a. Let X be a topological space. Then show that closed subset A of X is the disjoint union of its isolated point and its set of limit points.
 - b. Show that:
 - i. $\text{Int}(A') = (\overline{A})'$, for every subset A of a topological space;
 - ii. Boundary of a closed set A of a topological space, is no-where dense.
 - c. Let X be a second countable space. Then prove that any open base for X has a countable subclass which is also an open base.
- Q. 3 Attempt Any Two: 14**
- a. Prove that every separable metric space is second countable.
 - b. Prove that every compact Hausdorff space is Regular.
 - c. Define Compact Space. Prove that every closed subspace of compact topological space is compact.
- Q. 4 Attempt Any Two: 14**
- a. Show that Co-finite topological space is T_1 -space but not Hausdorff.
 - b. Show that any continuous image of a compact space is compact.
 - c. Prove that a one-to-one continuous mapping of a compact space onto a Hausdorff space is a homeomorphism.
- Q. 5 Attempt Any Two: 14**
- a. Give an example to show that a one-to-one continuous mapping of one topological space onto another need not be homeomorphism.
 - b. State and prove Urysohn's Lemma.
 - c. Show that every metric space is Normal.